Temperature Profiles in a Cylindrical Model Food During Pulsed Microwave Heating

H.W. Yang and S. Gunasekaran

ABSTRACT: Cylindrical 2%-agar gel samples were heated by pulsed and continuous microwave applications. The total microwave application time of 3 min was maintained for all experiments. Sample temperature was measured at various depths along the radial dimension to experimentally determine the internal temperature profile as a function of heating time. A local hot spot was observed at the center portion of the sample during the continuous microwave application. This hot spot was less significant during pulsed microwave applications, especially when longer intermittent power-off times were employed. An implicit finite-difference model was used to estimate temperature profiles within the sample during microwave heating. The estimated temperature profiles matched the experimental values well.

Key Words: agar gel, temperature profiles, pulsed microwave heating, finite-difference model, Lambert’s law

Introduction

Estimating temperature profiles within food and biological materials subjected to microwave radiation is very critical to the development of food processes and medical treatments that use microwave radiation. Regarding medical treatments, Ho and others (1971) reported microwave-heating patterns of human limbs (cylindrical models) associated with human arms and thighs exposed to a direct contact aperture source in which the waveguide and transmission lines are coupled through small apertures. They found differences in the heating patterns of cylinders representing arms and thighs with different aperture sizes. Kritikos and Schwan (1975) reported microwave-heating patterns of spheres representing human and animal heads of various sizes (radii). The computational time-temperature information that determines microbial safety and product quality can help avoid a trial-and-error approach to microwave process design (Mudgett 1986).

Finite-difference approximations have been used to obtain reasonable estimation of internal temperature profiles during microwave heating. Ohlsson and Bengtsson (1971) offered a 1-dimensional numerical solution for a finite slab to approximate the temperature profiles in meat blocks heated with microwave radiation. Nykvist and Decareau (1976) developed a 2-dimensional model for cylinders, representing meat roasts. Padua (1993) developed a temperature rise model for solidified cylindrical agar gels containing sucrose in terms of dielectric properties and total power absorbed. Barringer and others (1995) reported another 1-dimensional model representing thin slabs for temperature prediction of agar gels in terms of dielectric properties and various formulations of microwave power absorbed.

One of the major problems in microwave processing of foods is the non-uniform temperature distribution. Ohlsson and Rismans (1978) studied the non-uniform temperature distribution inside cylindrical and spherical meat and potato samples. Local hot spots at the sample center were investigated. Pulsed (intermittent) microwave application has been reported to result in lower energy requirements and improved temperature uniformity inside food materials compared to continuous microwave application. A series of studies by Shivhare and others (1992a, 1992b, and 1992c) showed that pulsed microwave drying of corn is more energy efficient than conventional hot air drying. Yongsawatdigul and Gunasekaran (1996a, 1996b) investigated the pulsed microwave drying of cranberries. They found that a pulsed application (30-s power-on, 150-s power-off) under vacuum (5.33 kPa) resulted in maximum drying efficiency. However, in these studies the temperature profiles (temperature uniformity) in the heated sample were not considered.

The objectives of this research were to: 1) estimate interior temperature profiles of a cylindrical model food (2% agar gel) heated by continuous and pulsed microwave application using implicit finite difference heat transfer model and 2) validate the model estimation with experimental data.

Mathematical Model

Implicit Finite–Difference Model

The microwaves in an oven cavity are multidimensional and can be described as polarization of transverse magnetic and transverse electric components. The microwave oven we used has a turntable operating at 14.3 rpm. Thus, a uniform field distribution within a heated sample can be expected. Two-dimensional finite element analysis of microwave power absorbed in cylindrical alginate gel samples by Lin and others (1995) indicate little variation of absorbed power along the axial direction. Much of the variation was only along the radial direction. Therefore, we used 1-dimensional finite difference model under the assumption that the microwave radiation is incident normal to the material surface (Padua 1993; Barringer and others 1995; Pangre and others 1991).

For a cylindrical geometry, the unsteady state (transient) differential equation may be obtained considering term-by-term difference approximation of the differential equation:

\[ \frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial z^2} + \frac{P}{\kappa} = \frac{\partial T}{\partial t} \]

(1)
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where, \( T \) = temperature, \( r \) = radial distance, \( z \) = axial distance, \( P \) = power generation, \( k \) = thermal conductivity, \( t \) = time, and \( \alpha \) = thermal diffusivity.

In the case of 1-dimensional heat transfer, the third term of Equation 1 on the left drops out and it becomes a second-order ordinary differential equation (Incropera and DeWitt 1996). Arpaci (1966) illustrated a finite difference (FD) formulation for cylindrical geometry. For the cylindrical geometry corresponding to a typical inner point, the first law of thermodynamics yields:

\[
\frac{P \rho C_p}{\tau} = q_{i-1}(R) \frac{dR}{2} \frac{d\tau}{2} + q_{i+1}(R) \frac{dR}{2} \frac{d\tau}{2}.
\]  

(2)

where \( R_i \) = the distance from the sample center, \( V_i \) = volume between \( R_i \pm dR/2 \), \( P_i \) = power generation in \( V_i \), \( \rho \) = density of the material, \( C_p \) = specific heat, \( \tau \) = time increment, \( dT_i \) = temperature difference at nodal point \( i \) between present time \( t \) and new time step \( t + \tau \). This method suffers from limitations on the selection of neighboring nodes for the preceding time \( t \). For the implicit method, the temperature of any node at \( t + \tau \) may be calculated from the knowledge of temperature at the same and neighboring nodes for the preceding time \( t \). This method guarantees a solution approach that is stable for all space and time intervals (Incropera and DeWitt 1996). Thus, we used the implicit method for estimating interior temperature profiles in the model food, 2% agar gel. The heat flux terms for the explicit and implicit schemes are listed in Table 1. For a typical boundary nodal point, the second term on the right side in Equation 2 depends on new temperatures of its adjoining nodes, which are generally unknown. Hence, to determine the unknown temperature at \( t + \tau \), the corresponding nodal equations must be solved simultaneously. The marching solution would then involve simultaneously solving the nodal equation at each time \( t, t + \tau, \ldots \) until the desired final time is reached. Compared to the explicit method, the implicit formulation has the advantage of being unconditionally stable, that is, it remains stable for all space and time intervals (Incropera and DeWitt 1996). The average temperature rise in a material within time \( t \) is expressed as the volume integral of the \( P(x) \) function. For a cylindrical sample in a uniform plane wave field:

\[
P_{\text{Total}}(x) = \int \phantom{\int} P(x) dV \int \phantom{\int} P_0 e^{-2\beta x} dx dz \int \phantom{\int} P_0 e^{-2\beta x} dx dz
\]  

(7)

where, \( x \) = depth or distance from sample center to the shell, \( \beta \) = attenuation constant in terms of dielectric properties, incident wavelength \( (\lambda_0) \), \( k' \) = dielectric constant and \( \kappa' \) = loss factor.

To calculate the incident power \( (P_0) \), the total power \( (P_{\text{Total}}) \) is expressed as the volume integral of the \( P(x) \) function. Integrating between limits and solving for \( P_0 \) gives:

\[
P_0 = \frac{\beta_0 V \gamma' \Delta T_{\text{air}}}{\pi Z (1 - e^{-2\beta R})}
\]  

(8)

The following expression can be used to calculate power absorbed \( (P_i) \) by each of the cylindrical shells in the sample (Padua 1993):

\[
P_i = \int \int \int P e^{-2\beta x} dx dz dx
\]  

(9)

where, the subscripts 1 and 2 refer to the outer and inner peripheries of the shell. In the case of pulsed microwave application, the power term for each node \( (P_i) \) is applied as heat generation term in Equation 2 during power-on periods and is zero for those time intervals during which the magnetron was not powered. This means that during every power-off period, there is no heat generation within the sample, and there is presumably only a conductive transfer occurring in the sample and a convective transfer across the boundary between the sample and ambient air.
Materials and Methods

Sample Preparation

Agar gels were prepared by dissolving 2% agar (Difco, Detroit, Mich., U.S.A.) in warm (approximate 40 °C) distilled water. The agar-water solution was heated until agar powder was totally dissolved and the gel solution was clear. Then the gel solution was poured into 400-mL beakers and cooled to room temperature into cylindrical solid samples (3.5 cm in radius and 7 cm in height). The solid samples were stored at 4 °C for 16 h to ensure uniform sample temperature.

Evaluation of Microwave Absorption

A laboratory microwave oven (Labotron 500, Zwag Inc., Epone, France) operating at 250-W continuous power output at 2450 MHz was implemented as the microwave-heating source. Since the agar gel sample is 98% water, its properties can be considered approximately equal to that of water (Padua 1993; Barringer and others 1995). Therefore, average temperature rise \( \Delta T_{av} \) of the gels was evaluated by measuring the temperature rise of a known volume (275 mL) of distilled water placed in the center of the microwave oven and heated for 3 min. The total power absorbed \( (P_{total}) \) was calculated using Equation 3. The power absorbed by each cylindrical shell \( (P_i) \) during each microwave power-on period was calculated using Equation 8 and 9.

Evaluation of Average Surface Heat Transfer Coefficient

The average surface heat transfer coefficient was determined by measuring the temperature of a cylindrical aluminum block of the same dimensions as the agar gel sample. The aluminum block was cooled to 4 °C and placed in the center of the microwave oven and allowed to warm by the ambient temperature in the oven (convection heat transfer). The temperature change (at center) was recorded every 10 s using a fiber-optic sensing system (MetricCor, Model 1400, Woodivelle, Wash., U.S.A.). The average surface heat transfer coefficient was calculated according to Rizvi and Mittal (1992) to be 42 W/m²·°C. Since thermal conductivity of the aluminum block is very high (very small Biot number), it was assumed that a lumped capacitance method could be used. In our experiment, air temperature was 22.7 °C, initial temperature of the aluminum block was 4 °C, surface area was 0.0194 m², mass of the block was 0.72 kg, and heat capacity was 0.930 kJ/kg·K.

Microwave Heating Processes

For each experiment, one agar gel sample was placed in the center of the microwave oven. For every sample, temperatures were measured across the horizontal middle plane at the radial distances of 0, 1, 2, and 3 cm for every min during the heating process (Figure 1a and b). Due to the use of a turntable inside the oven during heating, temperature measurement during heating using a fiber-optic sensing system may damage the sample. Therefore, temperature measurements were made by removing the sample from the oven.
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cavity. All measurements were made within 30 s at each radial distance using thermocouples inserted into the sample at the end of each heating step. Temperatures were recorded using a data logger (Model 34970A, Hewlett Packard, Beveryton, Oreg., U.S.A.). Analysis of temperature variation with time indicated that possible error introduced by temperature measurements per this procedure was less than 2°C.

Microwave power-on and -off times were adjusted to obtain different pulsing ratios (duty cycles) as follows (Figure 2): 1 (continuous), 2 (30 s power-on, 30 s power-off), and 3 (20 s power-on, 40 s power-off). The pulsing ratio, PR, is defined as:

\[
PR = \frac{t_{on} + t_{off}}{t_{on}}
\]  

(10)

where \(t_{on}\) and \(t_{off}\) = duration the microwave power is on and off per pulse, respectively. The total microwave power-on time was maintained 3 min for each PR. However, the total heating (process) times were 3, 6, and 9 min respectively for PR = 1, 2, and 3.

Data Analysis

The physical, dielectric, and thermal properties of 2%-agar gel reported by Barringer and others (1995) were used for calculation (Table 2). The attenuation constant (\(\beta\)) was calculated using Equation 5. The implicit transient finite-difference program was used to predict the internal temperature profiles of agar samples subjected to both continuous and pulsed microwave heating. The convection effect at the surface was also considered.

In the case of an infinite cylinder, which is initially at a uniform temperature and experiences a change in convective boundary condition with 1-dimensional transient heat conduction, the exact solution is given by (Incropera and DeWitt 1996):

\[
\frac{T_i - T_{\infty}}{T_i - T_{\infty}} = \sum_{n=1}^{\infty} C_n e^{rac{-2}{\zeta_n R}} I_0\left(\frac{\zeta_n R}{l_{on}}\right) J_1\left(\frac{\zeta_n R}{l_{off}}\right)
\]  

(11)

where \(F_0 = \frac{g_0 l_{on}}{R^2}\), the coefficient \(C_n\) is \(C_n = \frac{2}{\zeta_n J_1(\zeta_n l_{off}) - J_0(\zeta_n l_{on})}\), and the discrete values (eigenvalues) of \(\zeta_n\) are positive roots of the transcendental equation which are related by the Biot number (\(B_\lambda\)). The quantities \(I_1\) and \(J_0\) are Bessel functions of the first kind. For \(F_0 > 0.2\), the series analytical solution (Equation 12) can be approximated by single term:

Figure 3—Analytical and implicit finite-difference numerical solutions for temperature profiles in an infinite cylinder (see Table 3 for parameters values used for approximate analytical solution)

Figure 4—Comparison of temperature profiles according to the implicit finite-difference model (FD) and temperature-rise (TR) model by Padua (1993). The FD model was evaluated at shell thickness increments of 0.1, 0.15, and 0.2 cm

Table 3—Parameter values of an infinite cylinder used for validation of the implicit finite difference model.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Heat transfer coefficient, h (W/m²°C)</td>
<td>40</td>
</tr>
<tr>
<td>Radius, R (m)</td>
<td>0.03a</td>
</tr>
<tr>
<td>Height, Z (m)</td>
<td>0.7a</td>
</tr>
<tr>
<td>Thermal conductivity, k (W/m°C)</td>
<td>0.6</td>
</tr>
<tr>
<td>Biot numberb</td>
<td>2</td>
</tr>
<tr>
<td>Coefficient, (C_1)</td>
<td>1.34</td>
</tr>
<tr>
<td>Eigenvale, (\zeta_1)</td>
<td>1.6c</td>
</tr>
<tr>
<td>Initial uniform temperature, (T_{\infty}) (°C)</td>
<td>4</td>
</tr>
<tr>
<td>Ambient temperature, (T_{\infty}) (°C)</td>
<td>60</td>
</tr>
<tr>
<td>Time, s</td>
<td>100</td>
</tr>
<tr>
<td>Thermal diffusivity, (\alpha_{m}) (m²/s)</td>
<td>1.5*10⁻⁵</td>
</tr>
<tr>
<td>Fourier number, (F_0)</td>
<td>0.3d</td>
</tr>
</tbody>
</table>

aChosen to designate an infinite cylinder Z/R > 10
b\(B_\lambda = hR/k\)
cTable value from Incorpera and DeWitt (1996)
dChosen to designate conditions can be approximated \(F_0 > 0.2\)

Table 4—Measured sample mass-average temperature (°C) for PR = 1, 2, and 3 after 1, 2, and 3 min of total microwave application (TMA).

<table>
<thead>
<tr>
<th>TMA (min) Pulsing Ratio</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>12.6</td>
<td>13.0</td>
<td>12.8</td>
</tr>
<tr>
<td>2</td>
<td>22.6</td>
<td>23.3</td>
<td>23.7</td>
</tr>
<tr>
<td>3</td>
<td>34.6</td>
<td>33.1</td>
<td>32.6</td>
</tr>
</tbody>
</table>

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The implicit finite-difference scheme was validated first by considering a non-power generation case (that is \( P = 0 \)) and comparing the model prediction to the approximate analytical solution of Equation 13 for an example situation per parameter values listed in Table 3. The effective shell thickness for the proposed implicit finite-difference model including microwave power generation was then validated by comparing the results to those of the temperature raise model of Padua (1993) under the same conditions: continuous microwave heating for 15 s, sample radius = 3 cm; sample height = 8.6 cm, initial temperature = 22°C, power level = 1500 W and \( \Delta T_{av} = 4.4°C \). The predicted internal temperature profiles were calculated using the computational software, Engineering Equation Solver (Klein and Alvarado 1996). The \( x^2 \) test was used to accept or reject the hypothesis that measured and predicted temperatures were the same, at \( p \leq 0.05 \) (Bender and others 1989).

Results and Discussion

Model Validation

The analytical and numerical solutions for the infinite cylinder case matched very well, though the analytical solutions were slightly lower than the corresponding numerical values.

Figure 5—Experimental temperature profiles in cylindrical agar gel sample (3.5 cm in radius and 7 cm in height) after 1, 2, and 3 min of total microwave power application at different pulsing ratios (PR), (a), (b), and (c), respectively. The power incident is at the sample outer periphery.

Figure 6—Finite-difference model estimated (E) and measured (M) temperature profiles for pulsing ratios (PR) of 1, 2, and 3 at a radial distance of 0 cm (centerline).

Figure 7—Finite-difference model estimated (E) and measured (M) temperature profiles for pulsing ratios (PR) of 1, 2, and 3 at a radial distance of 1 cm from the centerline.
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(From the page). This is because we used only the first term of the series solution (that is, used only $\zeta_i$ instead of $\zeta_{n=1,2,3,...}$ in Equation 12 for the analytical solutions. The temperature estimation per the implicit FD model also agreed well with the solution of the temperature-rise model of Padua (1993). Different shell thickness increments used in FD model (0.1, 0.15, and 0.2 cm) did not affect the estimation significantly (Figure 4). These results indicated, as expected, that the implicit FD model was unaffected by the increment chosen. We chose 0.1 cm as the increment value for subsequent analyses.

**Temperature Profiles**

The measured sample temperature profiles after total microwave applications of 1, 2, and 3 min with PR = 1, 2, and 3 are shown in Figure 5 a, b, and c, respectively. The center (R = 0 cm) temperature of all samples was the highest among temperatures at all locations. This local hot spot became significant within a short time of continuous microwave heating (PR = 1). The measured temperature profiles showed a difference of 9.2°C between radial distances 0 and 1 cm at the end of 1 min with PR = 1. As the continuous microwave heating time increased, the local hot spot developed became more significant. The measured sample temperatures showed a difference of 13.7°C between radial distances 0 and 1 cm after 3 min with PR = 1. The uneven temperature distribution within continuous microwave heated food materials is undesirable for both process safety and quality.

Under pulsed microwave application (PR = 2 and 3), the uneven temperature distribution decreased substantially. After 3 min of a total microwave application (TMA) with PR = 2 and 3, the differences between temperatures at R = 0 and 1 cm were 5.8°C and 4.7°C, respectively. These results indicate that pulsed microwave application is preferable to continuous application in avoiding local hot spots. The higher the pulsing ratio, the better the temperature uniformity expected during microwave heating.

The convective heat transfer affects temperature distribution within the sample during heating. The air temperature inside the microwave oven was around 25°C. Once the sample surface temperature is higher than the ambient temperature, air convection becomes a cooling effect. The sample mass average temperatures after TMA of 1, 2, and 3 min were calculated to be within a narrow range for all PRs (Table 4). As the processing time increased, at higher PRs the average sample temperature tended to be lower. The differences in temperature profiles for the 3 PRs did not exceed 2°C after 3 min of TMA. Since the same TMA was used in all cases, it can be expected that the energy cost for all treatments are the same, and yet pulsed heating results in a more uniform temperature distribution than continuous heating.

The measured and FD model estimated temperature profiles for different pulsing ratios at sample radial distances of 0 and 1 cm are shown in Figure 6 and 7, respectively. The estimated temperature profiles along the sample center indicate that during the power-off periods, the temperature decreased. This implies that the direction of heat flux was outward because of the lower neighboring temperature. The estimated temperature profiles at a radial distance of 1 cm show that during power-off periods, temperature held steady or slightly increased because the heat flux direction from the center (higher center temperature) of the sample was inward (into 1 cm location). The differences between the estimated and corresponding measured temperatures can be expected because, as mentioned previously, temperatures were measured after a short delay (about 30 s) at the end of each microwave heating step. The pulsed applications for either PR = 2 or PR = 3 reduced the sample center temperatures significantly and increased temperatures at radial distances of 1, 2, and 3 cm (data for radial distances 2 and 3 cm are not shown). The conductive temperature redistribution during the rest times is a major reason for a more uniform temperature profile during pulsed heating compared to continuous microwave heating. The chi square value, $\chi^2$, was determined for the difference between estimated and measured temperatures for each microwave application. After 3 min of total microwave application, the estimated temperatures for PR = 1, 2, and 3 were not significantly different from the corresponding measured temperatures.

**Conclusions**

**Notations**

- $A_i$: surface area exposed to ambient air, m²
- $B_i$: Biot number
- $C$: coefficient of the analytical solution for an infinite cylinder
- $C_p$: specific heat capacity, kJ/kg°C
- $d_R$: increment of between two nodal points, m
- $F_0$: Fourier number: heat transfer coefficient, W/m²°C
- $J_0$: zero order Bessel function of first kind
- $J_1$: First order Bessel function of first kind
- $k$: thermal conductivity, W/m°C
- $P$: power generation with volume $V_i$, W
- $P_0$: incident power or power at the surface, W/m²
- $P_i$: power generation with volume $V_i$, W
- $P_{total}$: total microwave energy absorbed by the heated body, W
- PR: pulsing ratio of microwave application
- $q_{i-1}$: heat flux form an inner nodal point, W/m²
- $q_{i+1}$: heat flux form an inner nodal point, W/m²
- $r_i$: radial distance, m
- $R$: radius of the sample, m
- $R_i$: radial distance of a nodal point, m
- $t$: processing time, s
- $t_{off}$: microwave power-off time, s
- $t_{on}$: microwave power-on time, s
- $T_i$: temperature, °C
- $T_{air}$: temperature of ambient air, °C
- $T_{f_i}$: predicted temperature at nodal point $i$, present time, °C
- $T_{i-1}$: predicted temperature at the inner nodal point $i-1$, present time, °C
- $T_{i+1}$: predicted temperature at the outer nodal point $i+1$, present time, °C
- $T_{int}$: uniform initial temperature, °C
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\[ T_{n,i}: \] predicted temperature at nodal point \( i \), at new (next) time step, °C

\[ T_{n,i-1}: \] predicted temperature at the inner nodal point \( i-1 \), at new (next) time step, °C

\[ T_{n,i+1}: \] predicted temperature at the outer nodal point \( i+1 \), at new (next) time step, °C

\( V \): total volume of the sample, m³

\( V_i \): volume of sub-shell \( i \) for numerical model, m³

\( x \): distance from sample surface to the center, m

\( Z \): height of the sample, m; axial distance, m

\( \alpha \): thermal diffusivity, m²/s

\( \beta \): attenuation constant

\( \chi^2 \): chi square (statistical table) value

\( \delta \): loss angle, rad

\( \Delta T_{av} \): average temperature rise in the sample, °C

\( \eta \): series term involved in analytical heat transfer solution for an infinite cylinder

\( \kappa' \): dielectric constant

\( \kappa'' \): dielectric loss factor

\( \lambda_o \): incident wavelength, m

\( \mu \): angle in cylindrical coordinates, rad

\( \rho \): density of the sample, kg/m³

\( \tau \): time increment, s

\( \zeta \): eigenvalue or positive root of the transcendental equation

References


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